C- Scheme ECS (KT) C- Scheme winter 2025

Max. Marks: 80 Time: 3 hrs.

17/12/2025

N.B. : 1. Q1 is compulsory
2. Attempt any three questions from Q2 to Q6.
3. Figures to the right indicate full marks.

Q1. (a) Find the extremals of $\int_{x_1}^{x_2} \frac{(y')^2}{x^3}\,dx$. 5

(b) Verify Cauchy-Schwartz inequality for the vectors u = (2, 3, 1) and v = (3, 0, 4). 5

(c) Evaluate $\int_C \frac{3z-5}{z^2-4}\,dz$ where C is the circle $|z-2| = 1$. 5

(d) Calculate the coefficient of correlation from the following data. 5

X :	3	5	4	6	2
Y :	3	4	5	2	6

Q2. (a) Construct an orthonormal basis of R^3 using Gram-Schmidt process to $S = \{(3, 0, 4), (-1, 0, 7), (2, 9, 11)\}$. 6

(b) Fit a Poisson distribution to the following data. 6

No. of deaths	:	0	1	2	3	4
Frequencies	:	123	59	14	3	1

(c) Using the Rayleigh-Ritz method, solve the boundary value problem 8

$$\int_0^1 2xy - y^2 - (y')^2\,dx,\ y(0)=0,\ y(1)=0.$$

Q3. (a) Using residue theorem evaluate $\int_C \frac{z+3}{z^2+2}\,dz$ where C is the circle $|z| = 2$. 6

(b) Find the extremals of 6

$$\int_0^1 yy' + (y'')^2\,dx,\quad y(0)=0,\ y'(0)=1,\ y(1)=2,\ y'(1)=4.$$

(c) Find the singular value decomposition of $\begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix}$. 8

Q4. (a) Check whether following are subspaces of the space R^3. 6
(i) $W = \{(a, b, c) / a = 1, c = 1\}$
(ii) $W = \{(a, 0, 0) / a \in R\}$

(b) A r.v. X has the distribution 6

X:	0	1	2	3	4	5	6
p(x):	k	3k	5k	7k	9k	11k	13k

Find (i) k (ii) E(X) (iii) Var(X).

(c) Reduce the quadratic form $2x^2 - 2y^2 + 2z^2 - 2xy - 8yz + 6xz$ to canonical form and find its rank, index & value class. 8

Q5. (a) The equations of the two regression lines are $3x + 2y = 26$ and $6x + y = 31$. Find i) $\bar{x}$ and $\bar{y}$, 6

ii) coefficient of correlation between x & y.

(b) The marks obtained by 1000 students in an examination are found to be normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be more than 75. 6

(c) Evaluate using the Cauchy integral formula, 8

$$\int_C \frac{z^3+4}{z^4+4z^2}\,dz, \quad C:|z-2-2i|=3.$$

Q6. (a) Calculate rank correlation coefficient from the following data. 6

X :	40	42	45	35	36	39
Y :	46	43	44	39	40	43

(b) Evaluate $\int_C \frac{\cos \pi z}{(z-1)^3}\,dz$ where C is the circle $|z| = 2$. 6

(c) Expand $f(z) = \frac{1}{z^2-3z+2}$ as a Laurent's series in the regions 8

(i) $1 < |z-3| < 2$ (ii) $|z| < 1$.

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